

A Novel Ranking functions for Trapezoidal Intuitionistic Fuzzy Numbers – Area between Centroid of Centroids Method.

* Vinodkumar Vaddadi

Abstract

In developing society, now a days, optimized transports are more essential mainly in business logistics . This paper, main aim is to get shortest path by nearest nodes from origin to destination for reaching out the communication. In this way a novel ranking function with triangular intuitionistic fuzzy numbers of centroid of centroids. The ranking of intuitionistic fuzzy numbers is paramount in the decision making process in a fuzzy and uncertain environment. Proposed method gives a satisfactory result to all circumstances in logistics of business. Ranking function defined by various types of fuzzy numbers which include normal, generalized triangular intuitionistic fuzzy numbers along crisp numbers which are a special case of fuzzy numbers. Some numerical examples are calculated to illustrate the validity and advantages of proposed method.

Keywords: Intuitionistic Fuzzy Numbers, Centroid of a Triangle, Ranking function.

1. Introduction

In mathematics, transportation theory is concerned with the study of transportation and allocation of resources. In real life situations, there is a need to transport products from different sources to different destinations at minimum cost. The transportation problem is a special type of linear programming problem which deals with optimal allocation of goods from sources to destinations, considering factors such as capacities, demands, and transportation costs, with main objective being to minimize the transportation cost while satisfying supply and demand constraints. The general transportation problem was originally developed by Hitchcock [1], where parameters such as transportation cost, resource availability and demand exist in an uncertain manner. Present-day science and technology are characterized by complex processes and phenomena for which complete information are not always available. Uncertainty exists in most real-life situations, making it challenging to precisely determine transportation cost. The uncertainty is often represented by fuzzy numbers. Fuzzy set theory was introduced by Zadeh [2] in 1965, many

authors have proposed different methods for ranking fuzzy number and is well known field for handling uncertainty. In such cases, the concept of fuzzy transportation problem naturally arises. Fuzzy transportation models offer valuable insights for decision-making in logistics and networks supply.

Many researchers have utilized fuzzy set theory. Zimmermann [15] developed fuzzy linear programming which evolved into several fuzzy optimization methods for solving transportation problems. Kaur and Kumar [7] proposed a new method based on ranking function for solving FTP, assuming that the parameters of transportation problem are represented by generalized trapezoidal fuzzy numbers. Some of them have used the generalized fuzzy number. One such generalized form of fuzzy set is intuitionistic fuzzy set, proposed by Atanassov [16]. Intuitionistic fuzzy sets have elements with degrees of membership and non-membership in the sets, belonging to the real unit interval $[0,1]$, with their sum also belonging to the same interval. as well. Intuitionistic fuzzy sets aids decision makers in determining the degree of satisfaction, degree of

* Lecturer in Mathematics, RGUKT- Srikakulam, Andhra Pradesh, Email: vinodvaddadi85@gmail.com

dissatisfaction for quantity and in making decision about the level of acceptance and non-acceptance for transportation cost in any transportation problem. Therefore, intuitionistic fuzzy number would be more precise to use compared to fuzzy number for dealing with uncertain data

The ranking of intuitionistic fuzzy numbers is an important factor in the study of intuitionistic fuzzy set theory. In order to rank intuitionistic fuzzy numbers, one intuitionistic fuzzy number needs to be compared with the others by using ranking function. There are many ways to rank intuitionistic fuzzy numbers like membership ranking, non-membership ranking, composite and distance-based ranking.

The transportation problem is a fundamental issue in logistics and supply chain management, where the goal is to determine the most efficient way to transport goods from multiple suppliers to multiple consumers while minimizing cost, time, or other resources. Traditional methods often fall short when dealing with the complexities and uncertainties inherent in real-world scenarios. This thesis aims to address these gaps by employing ranking techniques within an intuitionistic fuzzy environment. Here are several reasons justifying the significance and need for this research:

In conclusion, the proposed research on transportation problems using ranking techniques in an Intuitionistic fuzzy environment is both timely and relevant. It addresses the limitations of traditional methods, enhances decision-making capabilities, and offers practical solutions for optimizing transportation systems under uncertainty.

Preliminaries

Fuzzy set Definition: Let U be a universe of discourse and A of U . A fuzzy set $\bar{A}$ of U is defined by a membership function $f_{\bar{A}}: U \rightarrow [0,1]$, where $f_{\bar{A}}(x)$ is the degree of x in $\bar{A}$ and is represented as $\bar{A} = \{(x, f_{\bar{A}}(x)) / x \in U\}$.

Intuitionistic Fuzzy Set: An Intuitionistic Fuzzy Set $\hat{A}$ in U is defined by $\hat{A} = \{(x, f_{\hat{A}}(x), g_{\hat{A}}(x)) / x \in U\}$, where $f_{\hat{A}}, g_{\hat{A}}$ are functions from U to $[0,1]$ represents degree of membership and non-membership of the element x in U and $0 \leq f_{\hat{A}}(x) + g_{\hat{A}}(x) \leq 1$, for all $x \in U$.

Intuitionistic Convex Fuzzy Set: An intuitionistic fuzzy set $\hat{A} = \{(x, f_{\hat{A}}(x), g_{\hat{A}}(x)) / x \in U\}$ of U is called an intuitionistic convex fuzzy set if membership function $f_{\hat{A}}(x)$ is fuzzy convex and non-membership function $g_{\hat{A}}(x)$ is fuzzy concave. i.e., $f_{\hat{A}}(\lambda x + (1 - \lambda)y) \geq f_{\hat{A}}(x) \wedge f_{\hat{A}}(y)$ and $g_{\hat{A}}(\lambda x + (1 - \lambda)y) \leq g_{\hat{A}}(x) \vee g_{\hat{A}}(y)$, where $x, y \in U, \lambda \in [0,1]$.

Intuitionistic Normal Fuzzy Set: An intuitionistic fuzzy set $\hat{A}$ is said to be normal fuzzy set if there exists at least two elements x, y in U such that $f_{\hat{A}}(x) = 1$ and $g_{\hat{A}}(y) = 1$.

Intuitionistic Fuzzy Number: An intuitionistic fuzzy set $\hat{A} = \{(x, f_{\hat{A}}(x), g_{\hat{A}}(x)) / x \in U\}$ of real line R is called an intuitionistic fuzzy number if it is satisfying the following.

1. $\hat{A}$ is intuitionistic fuzzy normal set
2. $\hat{A}$ is intuitionistic fuzzy convex set
3. $f_{\hat{A}}(x) \wedge g_{\hat{A}}(x)$ are piecewise continuous real valued function the support of $\hat{A}$ is bounded.

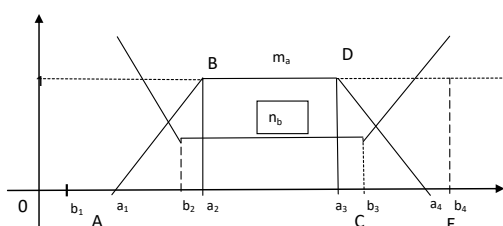
Trapezoidal Intuitionistic Fuzzy Number: A Trapezoidal Intuitionistic Fuzzy Number $\hat{A}$ with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ in this case we will give

$$f_{\hat{A}} = \begin{cases} 0, x < a_1 \\ \frac{x-a_1}{a_2-a_1}, a_1 \leq x \leq a_2 \\ 1, a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4}, a_3 \leq x \leq a_4 \\ 0, a_4 < x \end{cases} \text{ and } g_{\hat{A}} = \begin{cases} 0, x < b_1 \\ \frac{x-b_2}{b_1-b_2}, b_1 \leq x \leq b_2 \\ 1, b_2 \leq x \leq b_3 \\ \frac{x-b_3}{b_4-b_3}, b_3 \leq x \leq b_4 \\ 0, b_4 < x \end{cases}$$

In this regard if $b_2 = b_3 \wedge a_2 = a_3$, then we get a triangular intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq (b_2 = a_2 = a_3 = b_3) \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_4, b_4)$.

Ranking method: The mean of orthocenter of the membership and non-membership functions are used to find the rank of Trapezoidal Intuitionistic Fuzzy number and Triangular Intuitionistic Fuzzy number. Here we use the mean as the reference point of any geometrical figure and Centroid is useful as a ranking index.

Let $A = (b_1, a_1, b_2, a_2, a_3, a_4, b_4, m_a, n_b)$ is a generalized Trapezoidal Intuitionistic Fuzzy number where $b_1 < a_1 < b_2 < a_2 < a_3 < b_3 < a_4 < b_4$. And m_a is the maximum value of membership value and n_b is the minimum value of non-membership value.



In the above figure, the membership function of generalized trapezoidal Intuitionistic Fuzzy number divided into three triangles that are ΔABC , ΔBCD , ΔDCE .

To find the orthocenters of non-membership trapezoidal intuitionistic fuzzy number:

To find the orthocenter of ΔABC , $A(a_1, 0)$, $B(a_2, m_a)$, $C(a_3, 0)$ are vertices of ΔABC

The slope of $BC = \frac{m_a}{a_2 - a_3}$

$AA' \perp BC$ then therefore the slope of $AA' = \frac{a_3 - a_2}{m_a}$

Equation of the line AA' is $y = \left(\frac{a_3 - a_2}{m_a}\right)(x - a_1)$ eqn1.,

Next we find the slope of AB is $\frac{m_a}{a_2 - a_1}$

$CC' \perp AB$ then therefore the slope of $CC' = \frac{a_1 - a_2}{m_a}$

Equation of the line CC' is $y = \left(\frac{a_1 - a_2}{m_a}\right)(x - a_3)$ eqn2.,

Solve above equations we get,

The orthocenter of ΔABC is $O_1 = (x, y) = \left(a_2, \frac{(a_3 - a_2)(a_2 - a_1)}{m_a}\right)$

To find orthocenter of ΔBCD :

By the definition of the orthocenter,

In a right angled triangle the vertices of right angle is called orthocenter

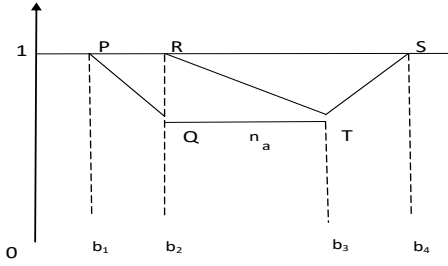
Therefore, the orthocenter of ΔBCD is $D = O_2 = (a_3, m_a)$

Similarly the orthocenter of right angled triangle ΔDCE is $C = O_3 = (a_3, 0)$

To find the centroid of all three orthocenters is G_1

$$G_1 = \Delta O_1 O_2 O_3 \\ = \left(\frac{a_2 + 2a_3}{3}, \frac{(a_2 - a_1)(a_3 - a_2) + m_a^2}{3m_a}\right)$$

To find the orthocenters of non-membership trapezoidal intuitionistic fuzzy number:



As shown in above diagram $\Delta PQR, \Delta QRT, \Delta RST$ are triangles of the non-membership function of trapezoidal and $P(b_1, 1)Q(b_2, n_a)R(b_2, 1)S(b_4, 1)T(b_3, n_a)$.

To find the orthocenter of ΔPQR :

By the definition of orthocenter, The orthocenter of ΔPQR is $O'_1 = (b_2, 1)$

To find the orthocenter of ΔQRT :

By the definition of orthocenter, the orthocenter of ΔQRT is $O'_2 = (b_2, n_a)$

To find the orthocenter of ΔRST :

$$\text{Slope of } RT = \frac{n_a}{b_3 - b_2}$$

$$SS' \perp RT \text{ then the slope of the line } SS' = \frac{b_2 - b_3}{n_a - 1}$$

$$\text{The equation of the line } SS' y = \left(\frac{b_2 - b_3}{n_a - 1} \right) (x - b_4) + 1 \text{ eqn.1,}$$

$$\text{Slope of } ST = \frac{n_a - 1}{b_3 - b_4}$$

$$RR' \perp ST \text{ then the slope of the line } RR' = \frac{b_4 - b_3}{n_a - 1}$$

$$\text{The equation of the line } RR' y = \left(\frac{b_4 - b_3}{n_a - 1} \right) (x - b_2) + 1 \text{ eqn.2,}$$

by solving eqn1 & eqn2 we get

$$\text{The orthocenter of } \Delta RST, O'_3 = (x, y) = \left(b_3, \left(\frac{b_2 - b_3}{n_a - 1} \right) (b_3 - b_4) + 1 \right)$$

Centroid

$$O'_1 O'_2 O'_3 = G_2 = \left(\frac{2b_2 + b_3}{3}, \frac{(2 + n_a)(n_a - 1) + (b_2 - b_3)(b_3 - b_4)}{3(n_a - 1)} \right)$$

Midpoint of Centroids of the triangles $\Delta O_1 O_2 O_3$ and $\Delta O'_1 O'_2 O'_3$

Centroid of

$$G_1 = \Delta O_1 O_2 O_3 = \left(\frac{a_2 + 2a_3}{3}, \frac{(a_2 - a_1)(a_3 - a_2) + m_a^2}{3m_a} \right)$$

Centroid of

$$G_2 = O'_1 O'_2 O'_3 = \left(\frac{2b_2 + b_3}{3}, \frac{(2 + n_a)(n_a - 1) + (b_2 - b_3)(b_3 - b_4)}{3(n_a - 1)} \right)$$

Ranking function is defined as product of x and y coordinates of the midpoint of the centers

$$\text{i.e., } Rank(A) = \frac{1}{36} \left[((a_2 + 2a_3 + 2b_2 + b_3)) \left(\frac{(a_2 - a_1)(a_3 - a_2)}{m_a} + \frac{(b_2 - b_3)(b_3 - b_4)}{(n_a - 1)} + m_a + n_a + 1 \right) \right]$$

Properties

1. Symmetrical Trapezoidal Intuitionistic fuzzy number:-

In general Trapezoidal Intuitionistic fuzzy number, $b_2 = a_2 \wedge b_3 = a_3$

Then Trapezoidal number become a symmetrical Trapezoidal Intuitionistic fuzzy number $(b_1, a_1, a_2, a_3, a_4, b_4)$

$$\text{Midpoint} = \left(\frac{a_2 + a_3}{2}, (a_2 - a_3) \left(\frac{(a_1 - a_2)}{6m_a} + \frac{(a_3 - a_4)}{6(n_a - 1)} \right) + \frac{m_a + n_a + 2}{6} \right)$$

$$Rank(A) = \left(\frac{a_2 + a_3}{2} \right) \left((a_2 - a_3) \left(\frac{(a_1 - a_2)}{6m_a} + \frac{(a_3 - a_4)}{6(n_a - 1)} \right) + \frac{m_a + n_a + 2}{6} \right)$$

2. Triangular intuitionistic fuzzy number:-

In general Trapezoidal Intuitionistic fuzzy number, $a_2 = a_3 \wedge b_2 = b_3$

Then Trapezoidal number become a symmetrical Trapezoidal Intuitionistic fuzzy number $(b_1, a_1, a_2, b_2, a_4, b_4)$

$$\begin{aligned} \text{Midpoint} &= \left(\frac{a_2 + b_2}{2}, \frac{m_a + n_a + 2}{6} \right) \text{Rank}(A) \\ &= \left[\frac{\frac{a_2 + b_2}{2} * m_a + n_a + 2}{6} \right] \end{aligned}$$

Put $m_a = 1$ and $n_a = 0$ then we get,

$$\begin{aligned} \text{Midpoint} &= \left(\frac{a_2 + b_2}{2}, \frac{1}{2} \right) \text{Rank}(A) \\ &= \left[\frac{\frac{a_2 + b_2}{2} * 1}{2} \right] \end{aligned}$$

3. $\text{Rank}(A) < \text{Rank}(B)$ iff $A < B$
4. $\text{Rank}(A) > \text{Rank}(B)$ iff $A > B$
5. $\text{Rank}(KA) = K \text{Rank}(A)$
6. $\text{Rank}(A+B) = \text{Rank}(A) + \text{Rank}(B)$.

Numerical Example: proposed Ranking function is follows

$$\begin{aligned} \text{Rank}(A) &= \frac{1}{36} \left[((a_2 + 2a_3 + 2b_2 \right. \\ &\quad + b_3)) \left(\frac{(a_2 - a_1)(a_3 - a_2)}{m_a} \right. \\ &\quad + \frac{(b_2 - b_3)(b_3 - b_4)}{(n_a - 1)} + m_a + n_a \\ &\quad \left. \left. + 1 \right) \right] \end{aligned}$$

This section gives some numerical example of the proposed method.

Example: Consider two trapezoidal Intuitionistic Fuzzy Numbers

$A = (1.1, 1.4, 1.7, 2.0, 2.3, 2.6, 2.9, 3.2)$

$B = (1.2, 1.8, 2.4, 3.0, 3.6, 4.2, 4.8, 5.4)$

Find the ranking value of above Tr.IFN we get,

$\text{Rank}(A) = 0.4$

$\text{Rank}(B) = 0.465556$

Since, $\text{Rank}(A) < \text{Rank}(B)$ therefore $A < B$

2. Conclusion

We discovered the area of centroids of membership Function and Non-membership function of Intuitionistic Fuzzy Numbers in this research and suggested a Area approach for

Ranking Intuitionistic Fuzzy Numbers based on the Centroid of Centroids. The suggested technique precisely orders intuitionistic fuzzy numbers. This method can be used to rank intuitionistic fuzzy numbers when taking various fuzzy optimization issues.

References:

1. Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & Mathematics with Applications*, 57(3), 413–419.
2. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87–96.
3. Atanassov, K. T. (1988). Review and new results on intuitionistic fuzzy sets. Preprint IM-MFAIS-1-88, Sofia.
4. Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 33(1), 37–45.
5. Atanassov, K. T. (2012). *On intuitionistic fuzzy sets theory* (Vol. 283). Springer.
6. Babu, S., Thorani, Y., & Shankar, N. R. (2012). Ranking generalized fuzzy numbers using centroid of centroids. *International Journal of Fuzzy Logic Systems*, 2(3), 17–32.
7. Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science*, 17(4), B141–B164.
8. Charnes, A., & Cooper, W. W. (1954). The stepping stone method for explaining linear programming calculations in transportation problems. *Management Science*, 1(1), 49–69.
9. Chen, S. M., & Tan, J. M. (1994). Handling multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets and Systems*, 67(2), 163–172.
10. Chhibber, D., Bisht, D., & Srivastava, P. K. (2019). Ranking approach based on incenter in triangle of centroids to solve type-1 and type-2 fuzzy transportation problem. *AIP Conference Proceedings*, 2061(1). AIP Publishing.
11. Chutia, R., & Saikia, S. (2018). Ranking intuitionistic fuzzy numbers at levels of decision-making and its application. *Expert Systems*, 35(5), e12292.
12. Dantzig, G. B. (1951). Application of the simplex method to a transportation problem. In *Activity Analysis of Production and Allocation* (pp. 359–373). Wiley.

13. Dinagar, D. S., & Palanivel, K. (2009). The transportation problem in fuzzy environment. *International Journal of Computational Mathematics*, 2, 65–71.
14. Ebrahimnejad, A. (2014). A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. *Applied Soft Computing*, 19, 171–176.
15. Garg, H., & Rani, M. (2018). An approach for solving intuitionistic fuzzy transportation problem using generalized trapezoidal intuitionistic fuzzy numbers. *Computers & Industrial Engineering*, 115, 512–519.
16. Grzegorzewski, P. (2003). Distances and orderings in a family of intuitionistic fuzzy numbers. In *EUSFLAT Conference* (pp. 223–227).
17. Grzegorzewski, P. (2004). Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. *Fuzzy Sets and Systems*, 148(2), 319–328.
18. Gupta, A., & Kumar, A. (2012). A new method for solving linear multi-objective transportation problems with fuzzy parameters. *Applied Mathematical Modelling*, 36, 1421–1430.
19. Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics*, 20, 224–230.
20. Kaur, A., & Kumar, A. (2011). A new method for solving fuzzy transportation problems using ranking function. *Applied Mathematical Modelling*, 35, 5652–5661.
21. Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problem using generalized trapezoidal fuzzy number. *Applied Soft Computing*, 12, 1201–1213.
22. Li, F. (2010). The information content of forward-looking statements in corporate filings—A naïve Bayesian machine learning approach. *Journal of Accounting Research*, 48(5), 1049–1102.
23. Wang, X., & Xin, J. (2005). An improved method for ranking intuitionistic fuzzy values. *IEEE Transactions on Fuzzy Systems*, 13(5), 688–692.
24. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338–353.
25. Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1(1).